Vector Calculus
Function Glossary

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The notation $f : X \to Y$ means the f is a function from a set X to a set Y.
Let $f : X \to Y$ and $g : Y \to Z$.
Let $A \subset X$ and $B \subset Y$.

- The *domain* of f is the set
$$\mathrm{dom}(f) = X.$$
- The *codomain* of f is the set
$$\mathrm{codom}(f) = Y.$$
- The *range* of f is the set
$$\mathrm{range}(f) = \{y \in Y \mid y = f(x) \text{ for some } x \in X\}.$$
- The *image* of A under f is the set
$$f(A) = \{y \in Y \mid y = f(a) \text{ for some } a \in A\}.$$
- The *preimage* of B under f is the set
$$f^{-1}(B) = \{x \in X \mid f(x) \in B\}.$$
- We say that f is *injective* if
$$f(x_1) = f(x_2) \quad \Rightarrow \quad x_1 = x_2.$$
- We say that f is *surjective* if
$$\text{for every } y \in Y \text{ there exists } x \in X \text{ such that } f(x) = y.$$
- We say that f is *bijective* if f is injective and surjective.
- The *composition* of f and g is
$$g \circ f : X \to Z \quad \text{given by} \quad g \circ f(x) = g(f(x)).$$
- The *identity function* on X is
$$\mathrm{id}_X : X \to X \text{ such that } \mathrm{id}_X(x) = x \text{ for all } x \in X.$$
- The *inverse function* of f is a function $g : Y \to X$ such that
$$g(f(x)) = x \quad \text{and } f(g(y)) = y \quad \text{for all } x \in X \text{ and } y \in Y\ .$$

  Equivalently, g is the inverse of f if
$$g(y) = x \quad \Leftrightarrow \quad f(x) = y.$$

  Equivalently, g is the inverse of f if
$$g \circ f = \mathrm{id}_X \quad \text{and} \quad f \circ g = \mathrm{id}_Y.$$